

Dimensional Bounds on Vircator Emission

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Vircators (Virtual Cathode Oscillators) are sources of short-pulsed, high power, microwave (GHz) radiation. An essentially dimensional argument relates their radiated power, pulse energy and oscillation frequency to their driving voltage and fundamental physical constants. For a diode of width and gap 10 cm and for voltages of a few hundred keV the peak radiated power cannot exceed $\mathcal{O}(30 \text{ GW})$ and the broad-band single cycle radiated energy cannot exceed $\mathcal{O}(3 \text{ J})$. If electrons can be accelerated to relativistic energies higher powers and radiated energies may be possible.

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Viractors (Virtual Cathode Oscillators) are sources of short-pulsed, high power, microwave radiation at multi-GHz frequencies. In a Viractor a sharply rising voltage pulse produces an intense pulse of field-emitted electrons. The electrons are attracted by a mesh anode. Most of them pass through the gaps in the anode, forming a “virtual cathode”. If the injected current density exceeds its space charge limit, a portion of the electron cloud reverses its motion as a result of its self-repulsion and the attractive force of the anode. The resulting oscillation radiates a short but intense pulse of microwaves. Maximizing net charge of the cloud and its acceleration maximizes the radiated power. Although there is an extensive literature^{1–11} and a wide variety of designs have been developed, comparatively little attention has been paid to fundamental limits on viractor performance and its scaling with driving voltage. These limits

Here we demonstrate upper limits to the radiated power of a viractor in a simple, essentially order-of-magnitude, model. Real viractors are more complicated—their radiation propagates in a waveguide rather than in free space as we assume, their electrons have a complicated distribution in phase space that must be calculated numerically, rather than our crude model described by a single length, density and velocity, the potential is also a complicated, numerically calculated, function, rather than a single static scalar as we assume, radiation by semi-relativistic electrons should be calculated numerically rather than from the result for dipole radiation (in the nonrelativistic limit) that we use.

Our model for radiated power has only one free parameter, the driving potential, and therefore leads to a simple result that is likely to bound the power produced in real, more complex, viractors. Our bound on the radiated energy also includes the size scale of the diode as a parameter. It is not possible to derive a single bound for more complete models and calculations of charges and currents in vacuum diodes^{12,13} (these authors did not calculate the emitted radiation), because they must be described by several independent parameters. A quantitative bound would require optimization over a several-dimensional parameter space; its origin would not be apparent and its numerical value would depend on detailed geometric assumptions. However, it would scale as our bounds do; only the numerical coefficients would be different. The origin and scaling of our simple bounds are transparent.

In the nonrelativistic limit our model consists of the following relations:

$$d = Qr \quad (1)$$

$$Q = nr^3e \quad (2)$$

$$V = \frac{Q}{r} \quad (3)$$

$$\alpha \equiv \frac{eV}{m_e c^2} \quad (4)$$

$$\omega = \omega_p \equiv \sqrt{\frac{4\pi n e^2}{m_e}}. \quad (5)$$

Here r is a length scale, Q is the charge of the electron cloud, n is its number density, V is a characteristic electrostatic potential, d is the magnitude of the oscillating dipole moment, ω_p is the nominal electron plasma frequency, which approximates the characteristic frequency ω of oscillation of the electron cloud and α is a dimensionless parameter describing the characteristic potential, and is of the same order as the imposed potential between cathode and anode. The remaining variables are the fundamental physical constants e , m_e and c .

We assumed that there is only one length scale r , that may be taken as the distance between cathode and anode. If the physical cathode is smaller, the electron cloud broadens to approximately this width. If the physical cathode and anode have areas $A \gg r^2$, as in a parallel-plate capacitor, the problem is essentially that of A/r^2 vircators in parallel, and our results should be multiplied by this ratio.

Combining these relations and using the relation¹⁴

$$P = \frac{1}{3} \frac{d^2 \omega^4}{c^3} \quad (6)$$

for the power P radiated by an oscillating electric dipole in free space, we find

$$P = \frac{(4\pi)^2}{3} \alpha^4 \frac{m_e^2 c^5}{e^2} = \frac{(4\pi)^2}{3} \alpha^4 \text{ 8.8 GW}. \quad (7)$$

The numerical coefficient is 3.3 for $\alpha = 0.5$, representative of vircators in practice. The expression for electric dipole radiation is applicable only for $\alpha \ll 1$, and is expected to be wrong for $\alpha > 0.5$

The electrons' oscillations rapidly dephase because of self-shielding that varies within the electron cloud¹⁵. As a result, the characteristic width of the pulse of radiation emitted by an instantaneous voltage pulse is $\sim 1/\omega$. The radiated energy

$$\mathcal{E} = \frac{1}{3} \frac{d^2 \omega^3}{c^3} = \frac{(4\pi)^{3/2}}{3} \alpha^{7/2} (m_e c^2) \left(\frac{r m_e c^2}{e^2} \right) = \frac{(4\pi)^{3/2}}{3} \alpha^{7/2} \left(\frac{r}{10 \text{ cm}} \right) \text{ 3 J}. \quad (8)$$

Although the instantaneous power is high, the radiated energy is small. The spectrum has the characteristic frequency

$$\nu = \nu_p = \sqrt{4\pi\alpha} \frac{c}{r} = \sqrt{\frac{\alpha}{\pi}} \left(\frac{1 \text{ cm}}{r} \right) 30 \text{ GHz.} \quad (9)$$

These numerical values are consistent with measurements in the nonrelativistic regime³.

Comparing the energy (8) to the electrostatic energy Q^2/r of the electron cloud leads to a radiation efficiency

$$\epsilon = \frac{(4\pi)^{3/2}}{3} \alpha^{3/2}, \quad (10)$$

valid only in the limit $\alpha \ll 1$. This is greater than measured³ efficiencies of vircator radiation for $\alpha \sim 0.5$, that are a few percent or less. This may be attributed to use of a rough approximation of the actual charge distribution and motion and to neglect of radiation reaction, which damps the motion of radiating charges, but which would involve a number of well-known paradoxes¹⁴.

Analogous estimates are possible in the ultra-relativistic limit. The relativistic form of the Larmor expression for the radiation by an accelerated charge, with acceleration parallel to the velocity (the same electric field E gives the electron cloud its relativistic velocity and further accelerates it) is

$$P = \frac{2}{3} \frac{Q^2 a^2 \gamma^6}{c^3}, \quad (11)$$

where \vec{a} is the acceleration, and γ the Lorentz factor. For $\vec{a} \parallel \vec{v}$

$$a = \frac{eE}{m_e \gamma^3}, \quad (12)$$

so the nonrelativistic result

$$P = \frac{2}{3} \frac{Q^2 e^2}{c^3 m_e^2} E^2 \quad (13)$$

is recovered.

Substituting $Q = \alpha m_e c^2 r / e$ and $E = \alpha m_e c^2 / (er)$ yields

$$P = \frac{2}{3} \alpha^4 \frac{m_e^2 c^5}{e^2}. \quad (14)$$

Aside from the numerical factor, this is the same result as Eq. 7 obtained in the nonrelativistic limit. The characteristic time scale of emission is now r/c , rather than $1/\omega_p$, and the corresponding radiated energy is

$$E = P \frac{r}{c} = \frac{2}{3} \alpha^4 \frac{m_e^2 c^4}{e^2} r. \quad (15)$$

The scaling with voltage is slightly different than that of the nonrelativistic result Eq. 8, but the dimensional factor is the same. The implied efficiency would be

$$\epsilon = \frac{2}{3}\alpha^2. \quad (16)$$

This would lead to the impossible result $\epsilon > 1$ in the ultra-relativistic limit $\alpha \gg 1$, a discrepancy that is explained by the neglect of radiation reaction.

The fact that the dimensional factors in the results for P and E are the same in the nonrelativistic and relativistic limits is unavoidable. There are only three dimensional quantities in the problem, once the electrostatic potential has been scaled to the electron rest mass, and therefore only one possible form for power and energy.

These are general limits on the performance of vircators, expressed in terms of fundamental physical constants, that are insensitive to details of design. They are consistent with measured performance; vircators readily emit powers of a few GW with driving potentials of a few hundred kV ($\alpha \approx 0.5$), but have not emitted orders of magnitude more. Although the emitted power is a steeply increasing function (proportional to the fourth power) of applied voltage in both nonrelativistic and ultra-relativistic regimes, the voltage than can be applied is limited by both available low-impedance power supplies (the voltage must rise rapidly in order than field emission not short out the potential before the full voltage is applied) and by the requirement that the voltage not be short-circuited by breakdown outside the diode.

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